Geometry

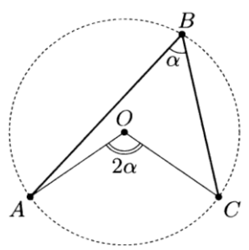

Figure 1:

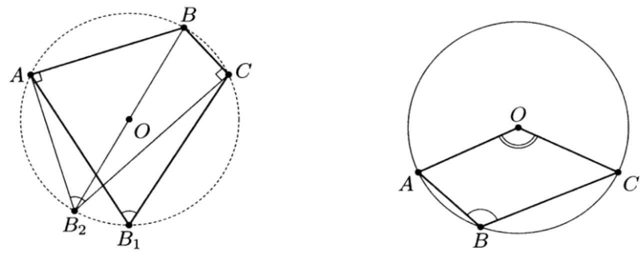

Figure 2:

The focus is on a few *nice* geometry theorems for inscribed angles. Refer to Figure 1 and figure 2 above.

i ***The inscribed angle in Figure 1 is half that of the central angle (or the arc angle) . (Do refresh the proof quickly)***

ii ***Figure 2 shows two angles B_1 and B_2 inscribed in the same arc and hence these angles are equal. Opposite angles in a cyclic quadrilateral add upto 180.***

iii ***Figure 2 on the right shows angle B + 1/2 angle O = 180.***

Some Illustrative problems

The above theorems can be used to solve interesting problems, we shall illustrate with a few examples.

i On the circle, there are four points A, B, C and D. What is the $\angle ADC$ if the $\angle ABC = \alpha$?

ii The side of the triangle opposite to its angle of 150 degrees is 10 cm. This triangle is circumscribed by a circle, find the radius of this circle.

iii Chords of the circle AC and BD intersect, the points M, N and K are the midpoints of the chords AB, BC and CD respectively. Prove that the $\angle BMN$ is equal to the $\angle NKC$.

iv Let AA_1 and BB_1 be the heights acute angled triangle ABC. Prove that the $\angle CA_1B_1$ is equal to $\angle CAB$.

The solutions are on the next page on the top of the page

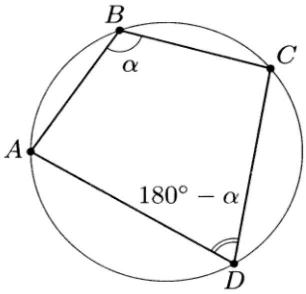

Figure 3:

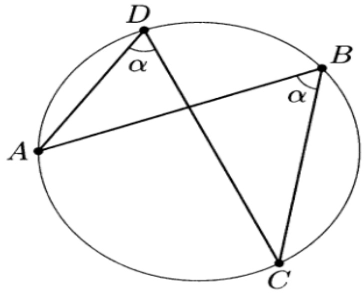

Figure 4:

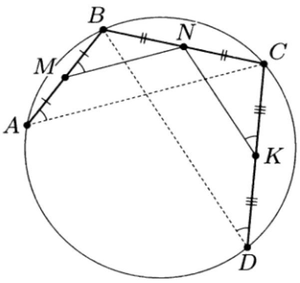

Figure 5:

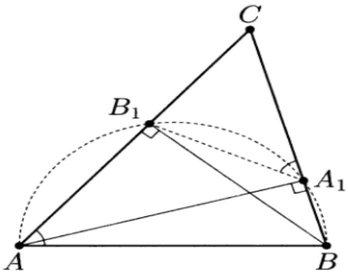

Figure 6:

Tasks

Task 1) The chords of the circle AD and BC intersect. The $\angle ABC$ is 50^o, the $\angle ADB$ is 80^o. Find the $\angle CAB$. *Task 2)* Points A, B and C lie on a circle. What is the $\angle ABC$ if the chord of the AC is equal to the radius of the circle? *Task 3)*Two angles are inscribed in the circle: $\angle ACB$ and $\angle A_1C_1B_1$. Prove that if they are equal, then $AB = A_1B_1$. Is the converse assertion true? *Task 4)* In the triangle ABC, the point O is the center of the circumscribed circle, $\angle A = \alpha$. Find the angle of the $\angle CBO$. *Task 5)* A hexagon is inscribed in a circle. Find the sum of the angles at its three non-adjacent vertices (see the figure 7 below). *Task 6)* Use a compass and ruler. A circle is given, construct a triangle with vertices on the circle when two of the angles of the triangle are specified values. *Task 7)* Let AB be the chord of a circle, C any point of this circle, M lie in the same half-plane with respect to the line AB as C. Prove that if the point M lies inside the circle, then the $\angle AMB$ is greater than the $\angle ACB$ and if outside the circle, then less. *Task 8)* On the circle, the points A and B are fixed, and the point C moves along one of the arcs AB. On what path does the center of the inscribed circle of triangle ABC move? *Task 9)* On the chord AB of the circle with center O the point C is taken. The circumcircle of the triangle AOC intersects the given circle at the point D. Prove that $BC = CD$. *Task 10)* Two circles intersect at points P and Q. A straight line intersects these circles sequentially at points A, B, C and D (see the figure 8 below). Prove that $\angle APB = \angle CQD$.

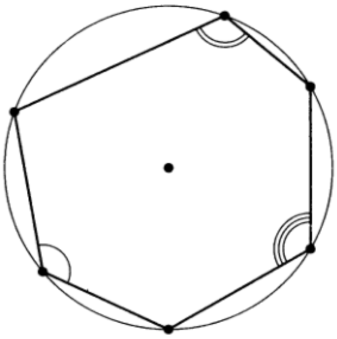

Figure 7:

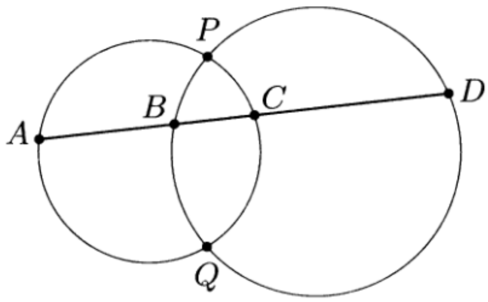

Figure 8: