

1. Problems (Introductory Number Theory)

Problems for the week

1 Problems leading to Solution of Linear Diophantine Equations

Each of the following problems requires you to form equations assuming different variables (maybe 2 or 3) using the given conditions in the problem. These equations can be manipulated to form a single linear Diophantine equation in two variables. Solve the resulting Diophantine equation applying Euclid's algorithm for the evaluation of GCD of the coefficients of the two variables.

- a) Refer to the notes taken in last club time for evaluation of GCD of two integers.
- b) If one particular solution of a Diophantine equation $ax + by = c$ is given by (x_0, y_0) , then all other solutions are $(x_0 + bk, y_0 - ak)$ when GCD of a and b is 1. Here $k \in \mathbb{Z}$. Also, consider the case when the GCD (of a and b) is not 1 but d , in this case if (x_0, y_0) is a particular solution then all other solutions are $(x_0 + (b/d)k, y_0 - (a/d)k)$ where again $k \in \mathbb{Z}$.

Problems

Find out the positive integer solutions by forming and solving linear inequation relevant to the statements given in the problem.

- a) There were 63 equal piles of plantain fruits put together and 7 single fruits. They were divided evenly among 23 travelers. What is the number of fruits in each pile? (*Ancient Indian puzzle - Mahaviracharya 850 A.D.*)
- b) If a cock is worth 5 coins, a hen 3 coins, and 3 chicks together 1 coin, how many cocks, hens, and chicks, totalling 100 can be bought for 100 coins? (*Ancient Chinese puzzle - 6th century*)
- c) Divide 100 into sum of two positive integers such that one is divisible by 7 and the other by 11.
- d) Find the number of men, women and children in a group of 20 persons if together they pay 20 coins, each men paying 3, each woman 2 and each child $1/2$.
- e) It was a strange lapse on the part of the cashier in a bank. He misread the cheque and handed out the amount of dollars in cents and the amount of cents in dollars. When the error was pointed out to him, he handed out a dollar, a ten cent coin, and a cent more. But the customer declared that he was still short of his due. The cashier pulled himself together, doubled the amount he had already given to the customer (that is, he handed an additional amount equal to the total amount he had given him previously), and so settled the transaction to everyone's satisfaction. What was the amount called for by the cheque?